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Current methods of spectral estimation can be broadly categorized under three main headings. One is classical power spectral density estimation which incorporates estimation of the autocorrelation function through lagged products and periodogram analysis and its variations. The second is power spectral density estimation based on modelling. This incorporates maximum entropy analysis, data extension using linear prediction and spectral estimation using ARMA models. The third is		

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20. ABSTRACT

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An Overview of Classical and High Resolution Spectral  
Estimation

SUMMARY of Invited Talk Presented at the  
L'Aquila Workshop on Digital Signal Processing

September 9-11, 1980

L'Aquila, Italy

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## An Overview of Classical and High Resolution Spectral Estimation

Current methods of spectral estimation can be broadly categorized under three main headings. One is classical power spectral density estimation which incorporates estimation of the autocorrelation function through lagged products and periodogram analysis and its variations. The second is power spectral density estimation based on modelling. This incorporates maximum entropy analysis, data extension using linear prediction and spectral estimation using ARMA models. The third is power spectral density estimation using adaptive windows which incorporates the method commonly referred to as the maximum likelihood (MLM) method. Classical power spectral density estimation typically has a time domain and a frequency domain interpretation. The time domain interpretation corresponds to estimating the autocorrelation function using lagged products of the given data. This procedure has the characteristic that for a given lag, the estimate of the correlation is unbiased and consistent as the length of the data record increases, but no matter how large the data record, the ends of the lagged products remain as bad estimates. Consequently, the lagged products are typically windowed to retain only those values that incorporated a "sufficient" number of points in the estimate. In the

frequency domain, classical power spectral density estimation is interpreted in terms of the periodogram which in essence corresponds to the magnitude squared of the Fourier transform of the data record. The procedure is particularly convenient with the FFT but has the property that it is not a consistent estimate. In particular, the variance of the estimate is approximately proportional to the spectrum being estimated. There are various ways of modifying the periodogram, including periodogram averaging, smoothing the periodogram or a combination of windowing the data and smoothing the periodogram.

As a means of introducing the other two categories of spectral estimation procedures, it is useful to focus on what can be thought of as two philosophical shortcomings of classical power spectral density estimation. In terms of the interpretation in the time or lag domain, classical PSD estimation provides reasonable estimates of the correlation function for certain lags, but implicitly assumes that the correlation function is zero outside the interval of the window. High resolution spectral analysis methods, such as MEM, which are based on modelling, can be thought of as providing more reasonable ways of extending the data or the autocorrelation function. In the frequency domain, classical PSD estimation corresponds to smoothing the periodogram with a

fixed window. Thus, the same smoothing is applied whether there are narrow lines close together or a broad spectral shape. As an alternative, use of an adaptive spectral smoothing window which changes its shape as it moves across the frequency band could be considered. This is in fact an interpretation of the maximum likelihood method of spectral analysis.

The most prominent procedure under the category of spectral estimation based on modelling is the maximum entropy method. Theoretically the method is formulated by assuming that the true autocorrelation function is known for a set of lags and it is then extended in such a way that the entropy, defined as the integral of the log spectral estimate, is maximized subject to the constraint that the spectral estimate matches the given autocorrelation values. It can be shown that this procedure is equivalent to all-pole modelling of the spectrum and in turn, all-pole modelling of a signal can be interpreted as a procedure of data extension using linear prediction. These two interpretations of maximum entropy also suggests several generalizations, one being the more general concept of a parametric model, such as a pole-zero or ARMA model. The second, suggested by the linear prediction interpretation of maximum entropy suggests extending the given data on the basis of a model and then applying classical

spectral analysis procedures to the extended data. Other variations on the maximum entropy method derive in part from the fact that in practice it is the data record which is given, not the autocorrelation values, and consequently the autocorrelation values must be estimated. There are several methods for doing this, for example, the correlation method, the covariance method, and the Burg recursion.

The maximum likelihood method of spectral estimation is derived from a different viewpoint, specifically that an attempt be made to smooth the underlying spectrum with an adaptive window which attempts to minimize leakage from neighboring frequencies. The method can be formulated by imagining filtering of the data with an FIR filter with the constraint that the filter frequency response have unity gain at the frequency at which the spectrum is to be estimated. Subject to this the window is designed to have minimum output power and it is the output power which is taken as the spectral estimate. When the resulting analysis was carried through, it can be shown that the resulting spectral estimate requires the autocorrelation function for the input sequence out to a number of lags equal to the length of the FIR filter. Since the autocorrelation values must be estimated from the given data record, this implies that the underlying FIR filter in the formulation of the method must be much shorter than the



length of the data record.

There are a wide variety of tradeoffs and comparisons between the three methods. With the classical method the underlying issue is principally one of resolution. The maximum entropy method is particularly effective at separating narrow lines in the spectrum, but can be very sensitive to the order of the model and effects such as peak splitting can occur. In addition, the maximum entropy method tends to follow peaks in the spectrum more closely than the valleys. The maximum likelihood method provides less resolution than the maximum entropy method as is clearly demonstrated through Burg's resistor averaging formula relating the two methods. On the other hand the procedure is more robust since it is not as susceptible to peak splitting as the MEM method. It has also been empirically demonstrated that for the MEM method, it is the area under a peak in the spectral estimate that is an indication of the signal power at that frequency, whereas with the MLM method it is the amplitude of the peak. For classical spectral analysis, both the peak amplitude and the area under the peak tend to be proportional to the signal power.

In summary, all three methods are derived on the basis of different assumptions and there are resulting characteristics and potential hazards particular to each of the methods.

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